1. Number Systems
## Common Number Systems

<table>
<thead>
<tr>
<th>System</th>
<th>Base</th>
<th>Symbols</th>
<th>Used by humans?</th>
<th>Used in computers?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimal</td>
<td>10</td>
<td>0, 1, … 9</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Binary</td>
<td>2</td>
<td>0, 1</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Octal</td>
<td>8</td>
<td>0, 1, … 7</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Hexa-decimal</td>
<td>16</td>
<td>0, 1, … 9, A, B, … F</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
### Quantities/Counting (1 of 3)

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
<th>Octal</th>
<th>Hexadecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>101</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>111</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>
## Quantities/Counting (2 of 3)

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
<th>Octal</th>
<th>Hexadecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1000</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>1001</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>1010</td>
<td>12</td>
<td>A</td>
</tr>
<tr>
<td>11</td>
<td>1011</td>
<td>13</td>
<td>B</td>
</tr>
<tr>
<td>12</td>
<td>1100</td>
<td>14</td>
<td>C</td>
</tr>
<tr>
<td>13</td>
<td>1101</td>
<td>15</td>
<td>D</td>
</tr>
<tr>
<td>14</td>
<td>1110</td>
<td>16</td>
<td>E</td>
</tr>
<tr>
<td>15</td>
<td>1111</td>
<td>17</td>
<td>F</td>
</tr>
</tbody>
</table>
### Quantities/Counting (3 of 3)

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
<th>Octal</th>
<th>Hexadecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>10000</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>17</td>
<td>10001</td>
<td>21</td>
<td>11</td>
</tr>
<tr>
<td>18</td>
<td>10010</td>
<td>22</td>
<td>12</td>
</tr>
<tr>
<td>19</td>
<td>10011</td>
<td>23</td>
<td>13</td>
</tr>
<tr>
<td>20</td>
<td>10100</td>
<td>24</td>
<td>14</td>
</tr>
<tr>
<td>21</td>
<td>10101</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>22</td>
<td>10110</td>
<td>26</td>
<td>16</td>
</tr>
<tr>
<td>23</td>
<td>10111</td>
<td>27</td>
<td>17</td>
</tr>
</tbody>
</table>

Etc.
Conversion Among Bases

• The possibilities:

- Decimal
- Octal
- Binary
- Hexadecimal
Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$
Decimal to Decimal (just for fun)

Decimal

Octal

Binary

Hexadecimal

Next slide…
125_{10} \Rightarrow 5 \times 10^0 = 5
2 \times 10^1 = 20
1 \times 10^2 = 100
\hline
125
Binary to Decimal

Decimal

Binary

Octal

Hexadecimal
Binary to Decimal

- Technique
  - Multiply each bit by $2^n$, where $n$ is the “weight” of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results
Example

Bit “0”

\[101011_2 \Rightarrow 1 \times 2^0 = 1\]
\[1 \times 2^1 = 2\]
\[0 \times 2^2 = 0\]
\[1 \times 2^3 = 8\]
\[0 \times 2^4 = 0\]
\[1 \times 2^5 = 32\]
\[\underline{43}_{10}\]
Octal to Decimal

Decimal

Octal

Binary

Hexadecimal
Octal to Decimal

• Technique
  – Multiply each bit by $8^n$, where $n$ is the “weight” of the bit
  – The weight is the position of the bit, starting from 0 on the right
  – Add the results
Example

\[724_8 \Rightarrow 4 \times 8^0 = 4\]
\[2 \times 8^1 = 16\]
\[7 \times 8^2 = 448\]

\[468_{10}\]
Hexadecimal to Decimal

Decimal

Octal

Binary

Hexadecimal
Hexadecimal to Decimal

• Technique
  – Multiply each bit by $16^n$, where $n$ is the “weight” of the bit
  – The weight is the position of the bit, starting from 0 on the right
  – Add the results
Example

$ABC_{16} \Rightarrow \begin{align*}
C \times 16^0 &= 12 \times 1 = 12 \\
B \times 16^1 &= 11 \times 16 = 176 \\
A \times 16^2 &= 10 \times 256 = 2560
\end{align*}$

$2748_{10}$
Decimal to Binary

- Decimal
- Octal
- Binary
- Hexadecimal
Decimal to Binary

• Technique
  – Divide by two, keep track of the remainder
  – First remainder is bit 0 (LSB, least-significant bit)
  – Second remainder is bit 1
  – Etc.
Example

\[ 125_{10} = \, ?_{2} \]

\[
\begin{array}{c|c}
2 & 125 \\
2 & 62 \\
2 & 31 \\
2 & 15 \\
2 & 7 \\
2 & 3 \\
2 & 1 \\
\end{array}
\]

\[
\begin{array}{c|c}
2 & 125 \\
2 & 62 & 1 \\
2 & 31 & 0 \\
2 & 15 & 1 \\
2 & 7 & 1 \\
2 & 3 & 1 \\
2 & 1 & 1 \\
\end{array}
\]

\[ 125_{10} = 1111101_{2} \]
Octal to Binary

Decimal

Binary

Octal

Hexadecimal
Octal to Binary

• Technique
  – Convert each octal digit to a 3-bit equivalent binary representation
Example

705_8 = ?_2

\[
\begin{array}{ccc}
7 & 0 & 5 \\
\downarrow & \downarrow & \downarrow \\
111 & 000 & 101
\end{array}
\]

705_8 = 111000101_2
Hexadecimal to Binary

- Decimal
- Octal
- Binary
- Hexadecimal
Hexadecimal to Binary

• Technique
  – Convert each hexadecimal digit to a 4-bit equivalent binary representation
Example

$10AF_{16} = ?_{2}$

\[
\begin{align*}
1 & \quad 0 & \quad A & \quad F \\
\downarrow & \quad \downarrow & \quad \downarrow & \quad \downarrow \\
0001 & \quad 0000 & \quad 1010 & \quad 1111
\end{align*}
\]

$10AF_{16} = 0001000010101111_{2}$
Decimal to Octal

Decimal → Octal
Hexadecimal

Binary → Hexadecimal
Decimal to Octal

- Technique
  - Divide by 8
  - Keep track of the remainder
Example

$1234_{10} = ?_8$

\[
\begin{array}{c}
8 & 1234 \\
8 & \underline{154} & 2 \\
8 & \underline{19} & 2 \\
8 & \underline{2} & 3 \\
8 & 0 & 2 \\
\end{array}
\]

$1234_{10} = 2322_8$
Decimal to Hexadecimal

- Decimal
- Octal
- Binary
- Hexadecimal

Diagram: Decimal to Hexadecimal flowchart.
Decimal to Hexadecimal

• Technique
  – Divide by 16
  – Keep track of the remainder
Example

$1234_{10} = ?_{16}$

\[
\begin{array}{c|cccc}
16 & 1234 & 2 & 13 = D \\
16 & 77 & 4 & 0 \\
16 & 4 & & \\
16 & 0 & & \\
16 & 0 & & \\
\end{array}
\]

$1234_{10} = 4D2_{16}$
Binary to Octal
Binary to Octal

• Technique
  – Group bits in threes, starting on right
  – Convert to octal digits
Example

\[ 1011010111_2 = ?_8 \]

\[ 1 011 010 111 \]
\[ \downarrow \downarrow \downarrow \downarrow \]
\[ 1 3 2 7 \]

\[ 1011010111_2 = 1327_8 \]
Binary to Hexadecimal

Decimal  Octal

Binary  Hexadecimal
Binary to Hexadecimal

• Technique
  – Group bits in fours, starting on right
  – Convert to hexadecimal digits
Example

$1010111011_2 = \ ?_{16}$

$1010111011_2 = 2BB_{16}$
Octal to Hexadecimal

Decimal

Binary

Octal

Hexadecimal
Octal to Hexadecimal

• Technique
  – Use binary as an intermediary
Example

$1076_8 = ?_{16}$

\[
\begin{array}{c}
1 \\
0 \\
7 \\
6 \\
\end{array}
\quad
\begin{array}{c}
001 \\
000 \\
111 \\
110 \\
2 \\
3 \\
E \\
\end{array}
\]

$1076_8 = 23E_{16}$
Hexadecimal to Octal

• Technique
  – Use binary as an intermediary
Example

$1F0C_{16} = ?_8$

\[
\begin{array}{ccccc}
1 & F & 0 & C \\
0001 & 1111 & 0000 & 1100 \\
1 & 7 & 4 & 1 & 4 \\
\end{array}
\]

$1F0C_{16} = 17414_8$
Exercise – Convert ...

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
<th>Octal</th>
<th>Hexadecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>1110101</td>
<td></td>
<td></td>
</tr>
<tr>
<td>703</td>
<td></td>
<td></td>
<td>1AF</td>
</tr>
</tbody>
</table>

Don’t use a calculator!

Skip answer  
Answer
Exercise – Convert …

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
<th>Octal</th>
<th>Hexadecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>100001</td>
<td>41</td>
<td>21</td>
</tr>
<tr>
<td>117</td>
<td>1110101</td>
<td>165</td>
<td>75</td>
</tr>
<tr>
<td>451</td>
<td>111000011</td>
<td>703</td>
<td>1C3</td>
</tr>
<tr>
<td>431</td>
<td>110101111</td>
<td>657</td>
<td>1AF</td>
</tr>
</tbody>
</table>

Answer
Common Powers (1 of 2)

- **Base 10**

<table>
<thead>
<tr>
<th>Power</th>
<th>Preface</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-12}$</td>
<td>pico</td>
<td>p</td>
<td>.000000000001</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>nano</td>
<td>n</td>
<td>.000000001</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>micro</td>
<td>µ</td>
<td>.00001</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>milli</td>
<td>m</td>
<td>.001</td>
</tr>
<tr>
<td>$10^3$</td>
<td>kilo</td>
<td>k</td>
<td>1000</td>
</tr>
<tr>
<td>$10^6$</td>
<td>mega</td>
<td>M</td>
<td>1000000</td>
</tr>
<tr>
<td>$10^9$</td>
<td>giga</td>
<td>G</td>
<td>1000000000</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>tera</td>
<td>T</td>
<td>1000000000000</td>
</tr>
</tbody>
</table>
Common Powers (2 of 2)

• Base 2

<table>
<thead>
<tr>
<th>Power</th>
<th>Preface</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^{10}$</td>
<td>kilo</td>
<td>k</td>
<td>1024</td>
</tr>
<tr>
<td>$2^{20}$</td>
<td>mega</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>$2^{30}$</td>
<td>Giga</td>
<td>G</td>
<td>1073741824</td>
</tr>
</tbody>
</table>

• What is the value of “k”, “M”, and “G”?

• In computing, particularly w.r.t. memory, the base-2 interpretation generally applies
Example

In the lab…
1. Double click on **My Computer**
2. Right click on **C:**
3. Click on **Properties**

\[
\frac{\text{Used space}}{2^{30}} = \frac{1,977,475,072 \text{ bytes}}{1,84GB}
\]
Exercise – Free Space

- Determine the “free space” on all drives on a machine in the lab

<table>
<thead>
<tr>
<th>Drive</th>
<th>Free space</th>
<th>GB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bytes</td>
<td></td>
</tr>
<tr>
<td>A:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>etc.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Review – multiplying powers

- For common bases, add powers

\[ a^b \times a^c = a^{b+c} \]

\[ 2^6 \times 2^{10} = 2^{16} = 65,536 \]

or...

\[ 2^6 \times 2^{10} = 64 \times 2^{10} = 64k \]
Binary Addition (1 of 2)

- Two 1-bit values

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A + B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

"two"
Binary Addition (2 of 2)

- Two $n$-bit values
  - Add individual bits
  - Propagate carries
  - E.g.,

\[
\begin{array}{c}
1 & 1 \\
10101 & 21 \\
+ 11001 & + 25 \\
101110 & 46
\end{array}
\]
Multiplication (1 of 3)

- Decimal (just for fun)

\[
\begin{array}{c}
35 \\
\times 105 \\
\hline \\
175 \\
000 \\
\hline \\
35 \\
\hline \\
3675 \\
\end{array}
\]
Multiplication (2 of 3)

- Binary, two 1-bit values

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A × B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Multiplication (3 of 3)

- Binary, two $n$-bit values
  - As with decimal values
  - E.g.,

$$
\begin{array}{c}
  1110 \\
  \times 1011 \\
\end{array}
\begin{array}{c}
  1110 \\
  1110 \\
  1110 \\
  0000 \\
  1110 \\
\end{array}
\overline{10011010}
$$
Fractions

- Decimal to decimal (just for fun)

\[
3.14 \Rightarrow 4 \times 10^{-2} = 0.04 \\
1 \times 10^{-1} = 0.1 \\
3 \times 10^0 = 3
\]

\[
\frac{3.14}{3} = 3.14
\]
Fractions

• Binary to decimal

10.1011 => \[ 1 \times 2^{-4} = 0.0625 \]
\[ 1 \times 2^{-3} = 0.125 \]
\[ 0 \times 2^{-2} = 0.0 \]
\[ 1 \times 2^{-1} = 0.5 \]
\[ 0 \times 2^0 = 0.0 \]
\[ 1 \times 2^1 = 2.0 \]
\[ \underline{2.6875} \]
Fractions

- Decimal to binary

3.14579

11.001001...

etc.
Exercise – Convert ...

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
<th>Octal</th>
<th>Hexadecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.8</td>
<td></td>
<td></td>
<td>C.82</td>
</tr>
<tr>
<td>101.1101</td>
<td></td>
<td>3.07</td>
<td></td>
</tr>
</tbody>
</table>

Don’t use a calculator!

Skip answer  Answer
### Exercise – Convert …

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
<th>Octal</th>
<th>Hexadecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.8</td>
<td>11101.110011…</td>
<td>35.63…</td>
<td>1D.CC…</td>
</tr>
<tr>
<td>5.8125</td>
<td>101.1101</td>
<td>5.64</td>
<td>5.D</td>
</tr>
<tr>
<td>3.109375</td>
<td>11.000111</td>
<td>3.07</td>
<td>3.1C</td>
</tr>
<tr>
<td>12.5078125</td>
<td>1100.10000010</td>
<td>14.404</td>
<td></td>
</tr>
</tbody>
</table>

Answer

![Smiley face](https://example.com/smile.png)
Thank you